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Magnetic Monopoles as Agents of Chiral Symmetry Breaking in U(1) Lattice Gauge Theory

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Abstract

We present results suggesting that magnetic monopoles can account for chiral symmetry breaking in abelian gauge theory. Full $U(1)$ configurations from a lattice simulation are factorized into magnetic monopole and photon contributions. The expectation $\langle\bar{\psi}\psi\rangle$ is computed using the monopole configurations and compared to results for the full $U(1)$ configurations. It is shown that excellent agreement between the two values of $\langle\bar{\psi}\psi\rangle$ is obtained if the effect of photons, which "dress" the composite operator $\bar{\psi}\psi$, is included. This can be estimated independently by measurements of the physical fermion mass in the photon background.

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$U(1)$ lattice gauge theory provides an ideal laboratory for studying the effects of monopoles. In $U(1)$, all non-perturbative effects are known to be caused by monopoles [1]. Further, there is an efficient procedure for identifying the magnetic current of monopoles in the link angle configurations generated in a $U(1)$ lattice gauge theory simulation [2]. Previously, two of the present authors have studied the heavy quark potential in $U(1)$ lattice gauge theory, and shown that in the confined phase, the value of the string tension calculated from monopoles agrees quantitatively with the full $U(1)$ string tension calculated directly from link angles [3].

In this paper, we turn our attention to the quenched chiral condensate in $U(1)$ lattice gauge theory, $\langle\bar{\psi}\psi\rangle$. We expect (and find) that this has a non-zero value in the confined phase, and vanishes in the deconfined phase. Our interest is in how the value of $\langle\bar{\psi}\psi\rangle$ calculated from the link angle configurations compares with the same quantity calculated from monopoles. Ascribing non-perturbative effects to monopoles would say that in the confined phase, a non-vanishing condensate will be induced by monopoles. Nevertheless, since a matrix element of a product of field operators is involved, this value should be renormalized by short distance perturbative effects *i.e.* photons, so to make a quantitative comparison, we also need to calculate this renormalization. Our method for doing so, described in more detail below, is to compute the ratio of renormalized to bare masses for a charge moving solely in the field of photons. Our principle result is that applying this renormalization factor to the monopole value of $\langle\bar{\psi}\psi\rangle$ yields the full $U(1)$ answer, to within statistical errors.

Let us now briefly describe the factorization of the link variables into photon and monopole parts. We start by resolving the $U(1)$ plaquette angles $\phi_{\mu\nu}$ into fluctuating and monopole parts,

$$\phi_{\mu\nu} = \phi'_{\mu\nu} + 2\pi m_{\mu\nu},$$

where $\phi'_{\mu\nu} \in (-\pi, +\pi]$. From the monopole term $m_{\mu\nu}$, we can define two currents, one electric, the other magnetic. The electric current is defined by $j_\mu = \nabla^-_\mu m_{\mu\nu}$. The magnetic current of monopoles is defined by $m_\mu = \nabla^{+*}_\mu m_{\mu\nu}$, where ${}^*m_{\mu\nu}$ is the dual of $m_{\mu\nu}$. The currents $j_\mu(m_\mu)$ reside on direct(dual) lattices.

We will compute the chiral condensate using the Banks-Casher formula [4] (see below) which involves the density of eigenvalues of the Dirac operator. Since the Dirac particle is electrically charged, the effect of monopoles on it must be represented by an electric vector potential. This four-vector potential derives from the electric current j_μ, and is analogous to the familiar three-vector potential $\vec{A}$ used for the case of a static monopole in the continuum. We have

$$A^{mon}_\mu(x) \equiv g\sum_y v(x-y)j_\mu(y), \tag{1}$$

where $g = 2\pi/e$ is the magnetic unit of charge, and v the lattice Coulomb propagator in Feynman gauge, satisfying $\nabla^+_\mu\nabla^-_\mu v(x-y) = -\delta_{x,y}$. By defining $U^{mon}_\mu(x) \equiv \exp(ieA^{mon}_\mu(x))$, we define the photon link by demanding that $\{U_\mu\}$ be factorized on each link:

$$U_\mu(x) \equiv U^{mon}_\mu(x)U^{phot}_\mu(x). \tag{2}$$

We can now examine the chiral condensate $\langle\bar{\psi}\psi\rangle$ separately on configurations $\{U_\mu\}^{mon}$ and $\{U_\mu\}^{phot}$, as well as the full $\{U_\mu\}$. In the quenched approximation $\langle\bar{\psi}\psi\rangle$ can be defined in the chiral limit via the eigenvalues of the Dirac operator [4]:

$$\langle \bar{\psi}\psi \rangle = \frac{\pi}{V}\rho(\lambda = 0), \tag{3}$$

where V is the lattice volume and $\rho(\lambda)d\lambda$ is the number of eigenvalues satisfying $\not{D}|n\rangle = i\lambda_n|n\rangle$ in the interval $(\lambda, \lambda + d\lambda)$, with $\not{D}[U_\mu]$ the staggered lattice fermion kinetic operator

$$\not{D}_{xy}[U_\mu] = \frac{1}{2}\sum_\mu (-1)^{x_1+\cdots+x_{\mu-1}} \left(U_\mu(x)\delta_{y,x+\hat{\mu}} - U^*_\mu(y)\delta_{y,x-\hat{\mu}} \right). \tag{4}$$

For finite V, $\rho(0)$ vanishes; therefore an extrapolation to the $\lambda \to 0$ limit must be made from the spectrum density at non-zero λ. In practice $\rho(\lambda)$ can be estimated by measuring the lowest $O(25)$ eigenvalues of $\not{D}$ per configuration using the Lanczos algorithm, and then using a binning procedure [5].

Calculations of the chiral condensate were performed with $U(1)$ configurations generated using the standard Wilson action on a 12^4 lattice. Three different values of the inverse coupling β were chosen: $\beta = 1.005, 1.010$, and 1.020. For each value of β, 5000 lattice updates were performed before beginning measurements to allow for equilibration. The next 4000 configurations were used for measurements skipping every 20 lattice updates to reduce correlations. For values $\beta \leq 1.010$, our results exhibited the breaking of chiral symmetry (i. e. $\langle \bar{\psi}\psi \rangle \neq 0$). These two values of β are known to correspond to the confinement phase as well [6]. For $\beta = 1.020$ the configurations were found to be in the chirally symmetric phase, which is coincident with the Coulomb phase.

Our results for the chiral condensate calculations for the chirally broken phase are presented in Figs. (1) and (2). We show both the full $U(1)$ calculations and the results from the monopole gauge field configurations obtained using Eq. (1). Using a linear fit and Eq. (3), the value of the chiral condensate was extracted from the 10 lowest values of $\rho(\lambda)$. The values for $\langle \bar{\psi}\psi \rangle$ are shown in Table I.

In Fig. (3) the results for the chiral condensate from full $U(1)$ fields and monopoles are shown for $\beta = 1.020$. For comparison, the results from the $\beta = 1.010$ monopole configurations are also included. Linear fits to $\rho(\lambda)$ yield a very small, but finite intercept. However, the values are found to be about a factor of 100 times smaller than the results in the broken phase. Thus, we are confident of being in the chirally symmetric phase for $\beta = 1.020$. For completeness, the $\beta = 1.020$ results are also included in Table I.

In Fig. (4) we show the eigenvalue spectrum calculated using the $\{U_\mu\}^{phot}$ background from both broken and symmetric phases. There is no signal of chiral symmetry breaking (note that we choose anti-periodic boundary conditions for the fermions in the temporal direction to avoid zero modes from near-plane wave solutions). This supports our conjecture that chiral symmetry breaking in this model can be ascribed entirely to monopoles.

Is it possible to account for the mismatch between $\langle \bar{\psi}\psi \rangle^{U(1)}$ and $\langle \bar{\psi}\psi \rangle^{mon}$? Since $\bar{\psi}\psi$ is a composite field operator, we expect it to be modified by quantum corrections independently of whether it acquires a vacuum expectation value. In perturbation theory, UV fluctuations in general result in the requirement for all field operators to be renormalized, and for composite operators to have an additional renormalization. This consideration leads us to the following hypothesis: the mismatch between the two condensate measurements is due to the rescaling of the $\bar{\psi}\psi$ operator by the fluctuations of the gauge fields contained in the $\{U_\mu\}^{phot}$ configurations; more concisely

$$\begin{aligned}\langle\bar\psi\psi\rangle^{mon} &= \langle(\bar\psi\psi)^{tree}\rangle^{mon}\\ \langle\bar\psi\psi\rangle^{U(1)} &= \langle(\bar\psi\psi)^{phot}\rangle^{mon},\end{aligned}\tag{5}$$

where $(\bar\psi\psi)^{phot}$ is the dressed operator which takes into account photon-like fluctuations, and the expectation value is taken in the monopole-only backgrounds. If the hypothesis is correct, then we can write

$$(\bar\psi\psi)^{phot} = Z(\bar\psi\psi)^{tree},\tag{6}$$

ie. the tree-level operator is multiplicatively enhanced by the photon fluctuations, the condensate data suggesting that the factor $Z \simeq 1.5$.

In support, we now describe an alternative and independent determination of Z, obtained by measuring the physical fermion mass m_R in the $\{U_\mu\}^{phot}$ background. On the assumption that the fluctuations in $\{U_\mu\}^{phot}$ are approximately Gaussian, then the dressing of the $\bar\psi\psi$ operator is given by a set of Feynman diagrams. Let $\Sigma(p)$ denote the complete set of 1PI diagrams describing corrections to the fermion two-point function. Then for the dressed fermion propagator we have

$$S_F^{phot}(p) = \frac{1}{i\not p + m_0 - \Sigma(p)} \simeq \frac{Z_2(a,\mu)}{i\not p + m_R},\tag{7}$$

where m_0, m_R denote bare and physical fermion masses respectively, Z_2 is a wavefunction rescaling which by analogy with perturbative QED we expect to be both gauge and cutoff-dependent, and the $\simeq$ symbol shows that the second equality holds only in the neighborhood of some subtraction point $i\not p = \mu$. Equation (7) may be rearranged to read

$$\Sigma(\mu) = (1 - Z_2^{-1}Z_m)m_0 + \mu(1 - Z_2^{-1})\tag{8}$$

with

$$m_R = Z_m(a,\mu)m_0.\tag{9}$$

Now, using the identity

$$\frac{d}{dm_0}\frac{1}{i\not p + m_0} = \frac{-1}{(i\not p + m_0)(i\not p + m_0)},\tag{10}$$

we see that the operation of differentiating with respect to m_0 is equivalent to a zero momentum insertion of a $\bar\psi\psi$ operator in a Feynman diagram. Therefore $-d\Sigma/dm_0$ is the set of 1PI diagrams, having one external ψ and one external $\bar\psi$, which describe corrections to $\bar\psi\psi$, ie.

$$(\bar\psi\psi)^{phot} = Z_2\left(1 - \frac{d\Sigma(\mu)}{dm_0}\right)(\bar\psi\psi)^{tree} \equiv Z(\bar\psi\psi)^{tree},\tag{11}$$

ie.

$$ZZ_2^{-1} = 1 - \frac{d\Sigma}{dm_0},\tag{12}$$

where $Z(a, \mu)$ is the rescaling factor we seek, and the factor Z_2 is included because in the diagrammatic approach matrix elements are evaluated with dressed fermion propagators on the external legs. Combining (8) with (12), we find

$$Z = Z_m = \frac{m_R}{m_0}. \tag{13}$$

Hence a measurement of m_R in the $\{U_\mu\}^{phot}$ background as a function of m_0 gives an independent estimate of Z.

This argument is essentially the same as that used to establish $Z_{\bar{\psi}\psi} = m_0/m_R$ in standard renormalized perturbation theory, where $(\bar{\psi}\psi)_R = Z_{\bar{\psi}\psi}(\bar{\psi}\psi)$ is the renormalized operator whose Green functions with other renormalized fields and operators are finite [7]. Here we make no attempt to define a renormalized operator, since bare quantities are evaluated in a lattice simulation, but rather use the same formalism to quantify the effects of operator enhancement by quantum fluctuations. Note that we have not specified the subtraction point defining Z very precisely, and have assumed that Z is independent of m_0. From experience in perturbation theory, we expect that numerically the most significant contribution to Z, of $O(\ln a)$, comes from short wavelength fluctuations, and that Z is relatively insensitive to the details of the subtraction.

To measure the physical fermion mass in the $\{U_\mu\}^{phot}$ background, we performed calculations of the fermion propagator starting with the same configurations used to compute $\rho(\lambda)$. Since the fermion propagator is not gauge invariant, it is first necessary to fix a gauge. Although a gauge transformation of $\{U_\mu\}^{phot}$ is not strictly a symmetry of the full theory, this procedure is justified since $\bar{\psi}\psi$, and by hypothesis m_R, are gauge invariant. In this work we have used Landau gauge [8] and extracted the lattice fermion timeslice propagator

$$C_f(x_4) = \mathrm{Re} \sum_{x_1, x_2, x_3 even} \left\langle (\not{D}[U_\mu] + m_0)^{-1}_{0,x} \right\rangle^{phot} \tag{14}$$

using a conjugate gradient routine, for bare mass values $m_0 a = 0.1, 0.09, \ldots, 0.04$. The restriction to spatial sites an even number of lattice spacings from the origin in each direction improves the signal [9]. Note that we have not attempted to fix the residual gauge freedom as in [9], but have instead relied on the fluctuations in $\{U_\mu\}^{phot}$ being intrinsically small (see below). The physical mass m_R can now be estimated by fitting C_f to the following functional form:

$$C_f(x_4) = A \left(\exp(-m_R x_4) + (-1)^{x_4} \exp(-m_R(L - x_4)) \right), \tag{15}$$

where L is the lattice size in the time direction. We used 100 configurations from each of the three β values previously studied. In order to take into account correlations between renormalized mass estimates due to using the same configurations for different bare masses we applied a bootstrap fitting routine.

To determine Z we plotted the mass obtained from C_f against the input bare mass, and fitted the results to a linear form:

$$m_R(m_0) = Z\, m_0 + b. \tag{16}$$

Fig. (5) shows a graph of $m_R(m_0)$ from our simulation for $\beta = 1.010$. The graphs for the other two values of β are very similar. The results for Z and b from the linear fits for each

value of β are shown in Table II. From the table it can be observed that the value of Z is a smooth function through the phase transition. Unexpected in our results were the small non-zero values of the intercept. This will be discussed briefly below.

Finally, we compare our value of Z calculated using $\{U_\mu\}^{phot}$ with the value observed in the chiral condensate calculations. We define the renormalization factor from the chiral condensate to be

$$Z_{\bar{\psi}\psi} = \frac{\langle\bar{\psi}\psi\rangle^{U(1)}}{\langle\bar{\psi}\psi\rangle^{mon}}, \tag{17}$$

and present the comparison in Table III. The two independent determinations of the renormalization factor show excellent agreement. This answers the question asked earlier: Is it possible to account for the mismatch between $\langle\bar{\psi}\psi\rangle^{U(1)}$ and $\langle\bar{\psi}\psi\rangle^{mon}$? Our results demonstrate that the mismatch is simply operator renormalization due to the photons.

As yet we have no satisfactory explanation for the non-zero intercepts in Table II, implying a small breaking of chiral symmetry in the photon-only configurations. It could be either a finite volume effect, or perhaps a residual gauge freedom associated with Gribov copies and the consequent non-uniqueness of the Landau gauge, or perhaps due to spatially constant gauge field modes. The only effect that we have investigated quantitatively is that of constant modes. We repeated our previous calculations at $\beta = 1.005$, this time performing a global gauge transformation to remove constant modes from the gauge fields. This was done in the following way: The full gauge fields were shifted by a direction dependent constant,

$$A'_\mu(x) = A_\mu(x) + c_\mu, \tag{18}$$

where c_μ is chosen so $\sum_x A'_\mu(x) = 0$. The new gauge fields A'_μ were used to recompute Z and the chiral condensates. In repeating the calculations with this gauge fixing we observed no measurable difference in the results.

Although we are not able to explain the small non-zero intercepts, the overall agreement of the slope with the value of Z required to explain the condensate results is extremely satisfying, and supports our hypothesis on the role of the residual photon fluctuations enhancing the $\bar{\psi}\psi$ operator.

An obvious direction in which to extend this analysis is the exploration of chiral symmetry breaking in non-abelian gauge theories following the identification of monopole networks after abelian projection. Studies of this kind have already appeared [10]; it is interesting to note that in the maximal abelian gauge used, there is a similar mismatch in the measured values of $\langle\bar{\psi}\psi\rangle$ between full and monopole-only configurations. It would be interesting to check whether this effect could be accounted for by the operator renormalization described here.

Finally note that even though this argument assumed a split of the background gauge field into Gaussian fluctuations, described by Feynman diagrams, and non-perturbative monopole-only configurations, the idea of classifying effects into those which rescale a local operator and those which lead to a non-vanishing expectation value for the operator may be generalized. For instance, it may in principle be possible to extend the analysis by factoring configurations in a scale dependent fashion, including only large monopole loops in the monopole part, and seeing if small monopole loops simply renormalize $\bar{\psi}\psi$ by comparing

the Z factors measured in two different ways. This may prove to be an effective probe of the scale at which non-perturbative effects manifest themselves. The scale dependence of monopole contributions to the string tension was studied in [11].

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TABLES

TABLE I. The values of $\langle\bar{\psi}\psi\rangle = \frac{\pi}{V}\rho(0)$ obtained from linear fits to $\rho(\lambda)$.

β	$\langle\bar{\psi}\psi\rangle^{U(1)}$	$\langle\bar{\psi}\psi\rangle^{mon}$
1.005	0.165(3)	0.105(1)
1.010	0.138(2)	0.089(2)
1.020	0.002(1)	0.0009(6)

TABLE II. Results of the fits $m_R(m_0)$ (cf. (16))

β	Z	b
1.005	1.570(4)	0.0124(3)
1.010	1.561(5)	0.0126(3)
1.020	1.524(4)	0.0115(3)

TABLE III. Comparison of our two determinations of the renormalization factor Z; The column labeled Z_m comes from the photon configurations via Eq. (16), and the column labeled $Z_{\bar{\psi}\psi}$ comes from the chiral condensates via Eq. (17)

β	Z_m	$Z_{\bar{\psi}\psi}$
1.005	1.570(4)	1.570(30)
1.010	1.561(5)	1.550(40)

FIGURES

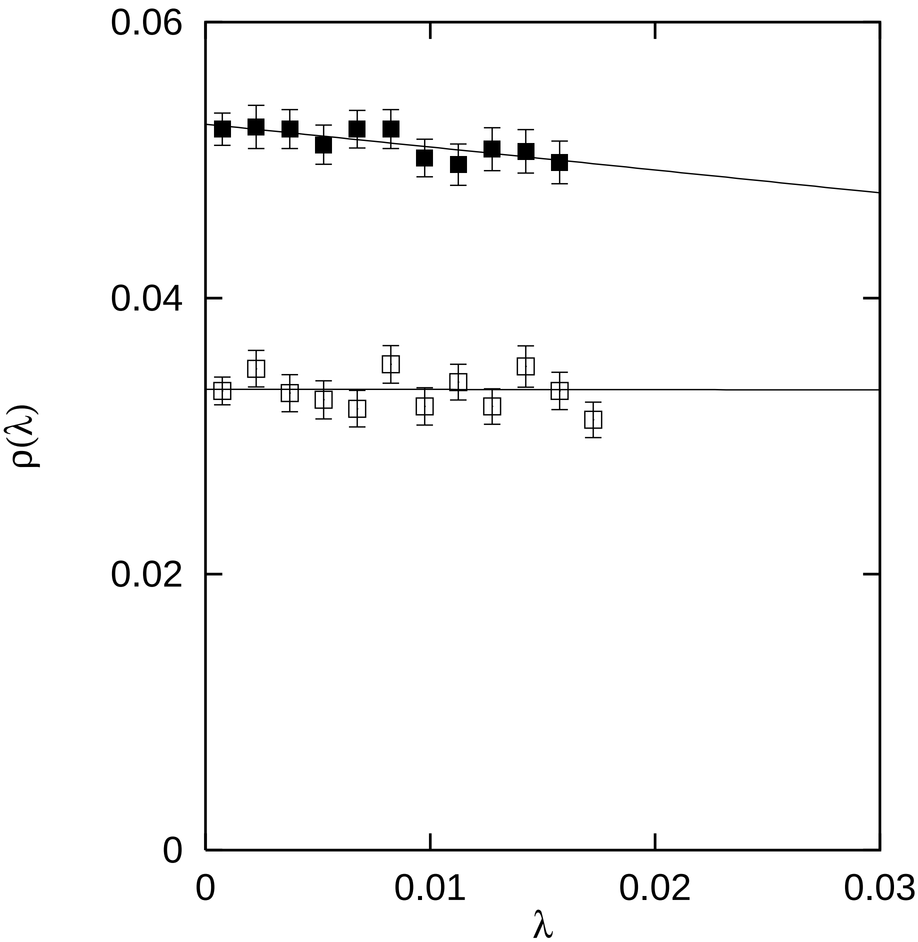

FIG. 1. The spectral density function for $\beta = 1.005$ from full $U(1)$ fields (solid squares) and monopoles (open squares).

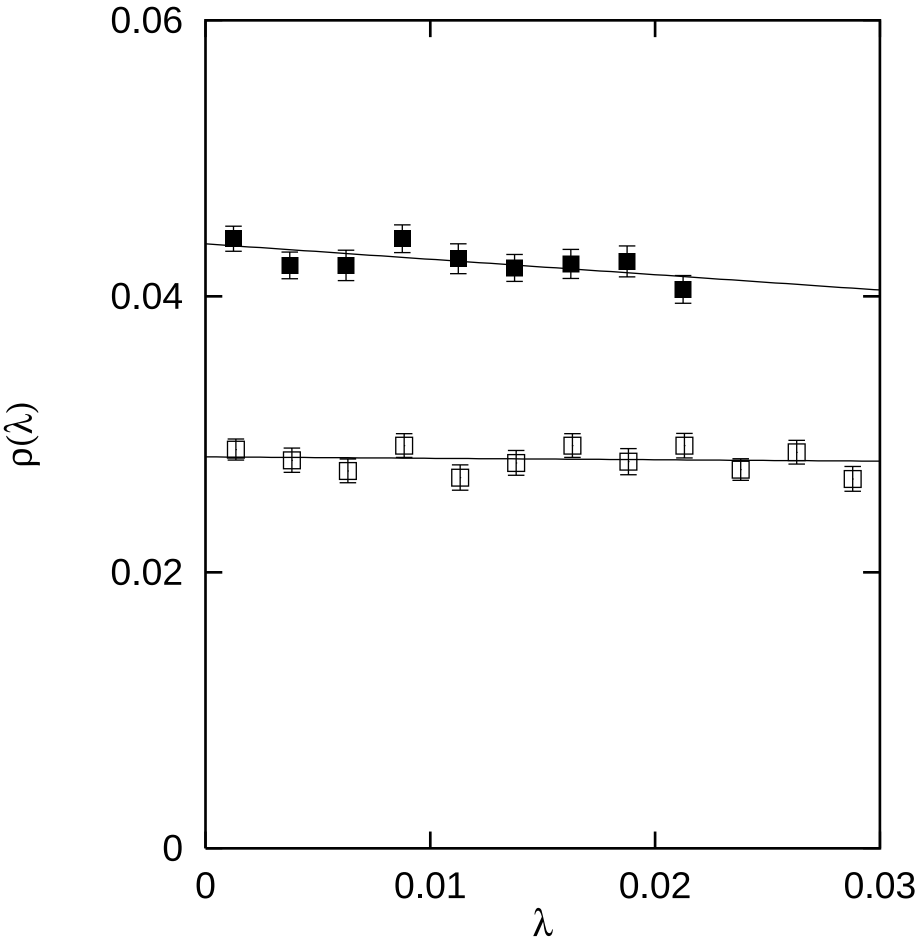

FIG. 2. The spectral density function for $\beta = 1.010$ from full $U(1)$ fields (solid squares) and monopoles (open squares).

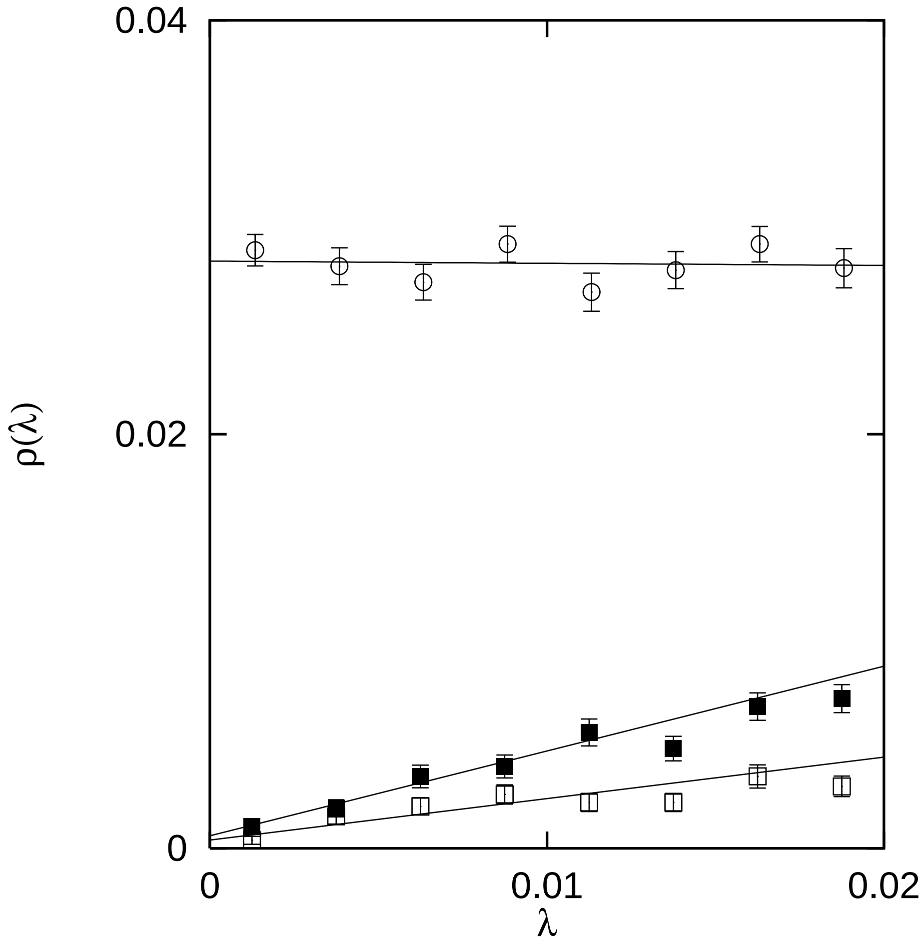

FIG. 3. The spectral density function for $\beta = 1.020$ from full $U(1)$ fields (solid squares) and monopoles (open squares). For comparison the results from monopoles for $\beta = 1.010$ (open circles) are also plotted.

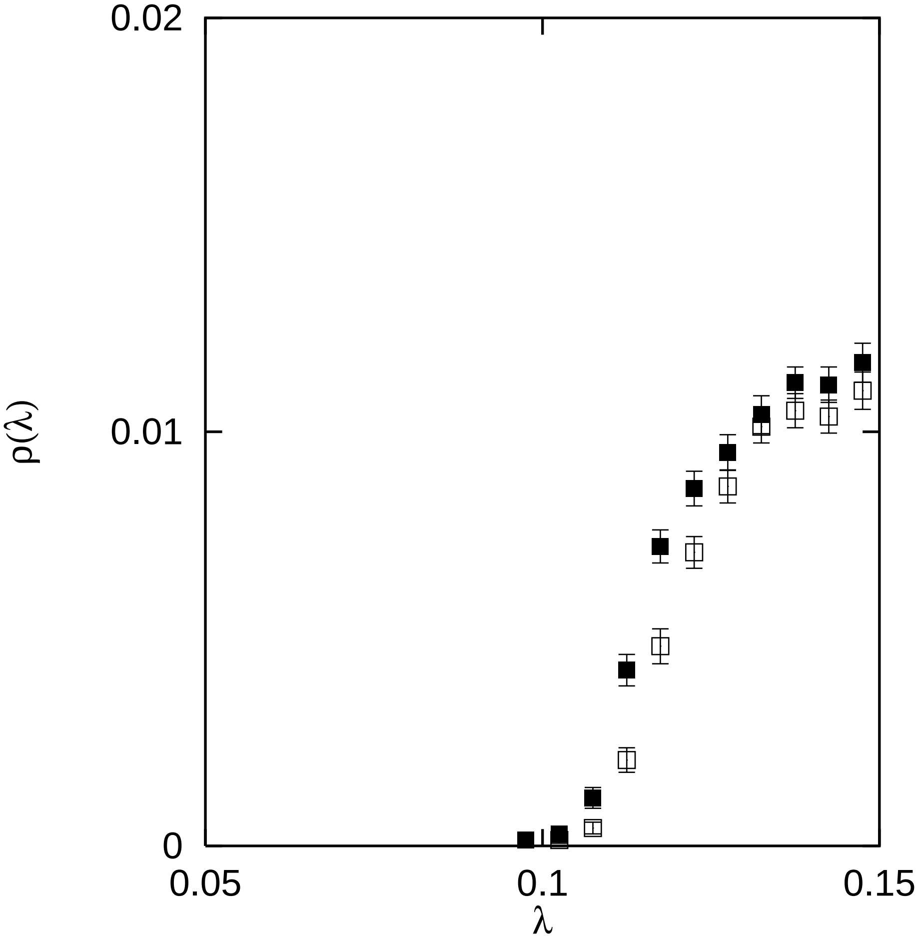

FIG. 4. The spectral density function from the photon background $\{U_\mu\}^{phot}$ for $\beta = 1.005$ (solid squares) and $\beta = 1.020$ (open squares).

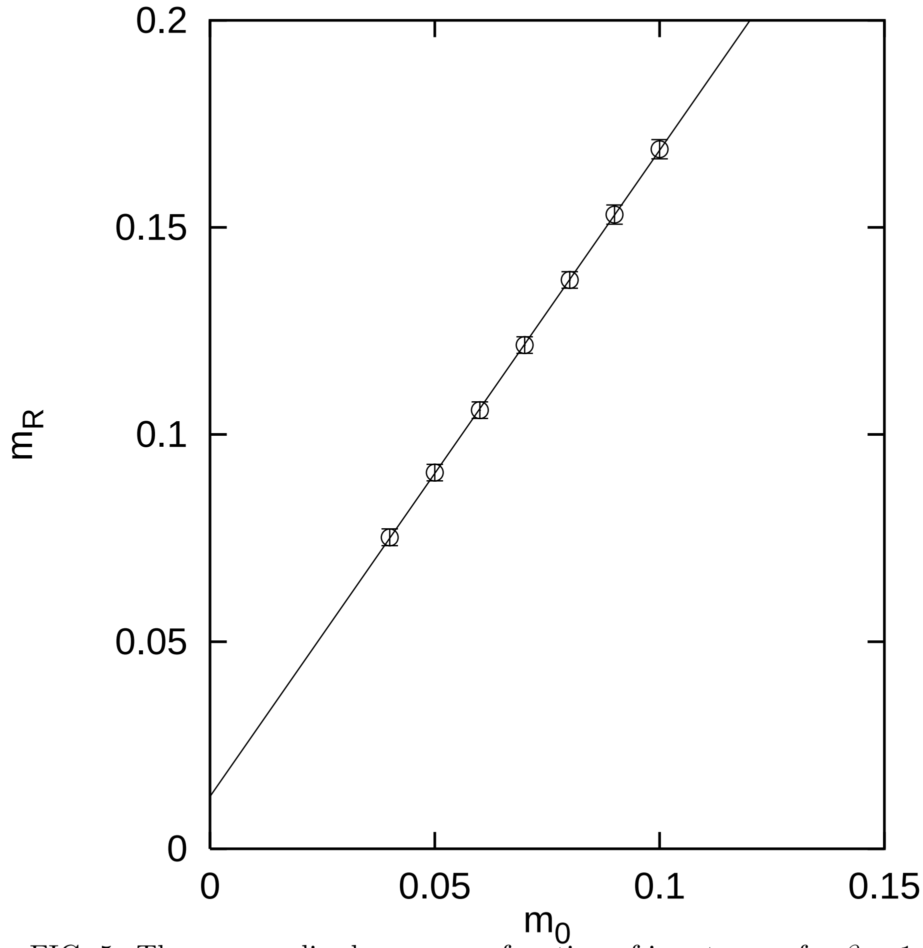

FIG. 5. The renormalized mass as a function of input mass for $\beta = 1.010$ calculated using the photon propagator as defined in Eqs. (14) and (15).